

We are heading toward Statistical Inference

Example of what we're going to do (eventually):

We suspect the average IQ of all ENC students is 120:

$$\mu = 120$$

We take a random sample of 30 ENC students and find the average of that sample to be 124:

$$\bar{X} = 124$$

Does $\bar{X} = 124$ support $\mu = 120$ or not?

Essentially we are going to ask:

What is the probability we get $\bar{X} = 124$ if $\mu = 120$?

Definitions/Terminology

Added to the Vocab List

Probability: A measurement of the likelihood of the occurrence of some chance event.

Outcome: A possible result of an experiment.

Sample Space: The set of all possible outcomes.

Event: A subset of the sample space, or a set of outcomes.

Simple Event: An event that has only one element. (Cannot be broken down any further.)

Examples of experiments

- a. Toss a coin one time and observe the side that shows.
- b. Roll a die once and observe the up side.
- c. Roll two dice and observe the pair of numbers that show.
- d. Buy a lottery ticket where 5 marbles are selected from 30 marbles.

Some rules of probability

- 1) $P(\text{impossible event}) = 0$
- 2) $P(\text{guaranteed event}) = 1$
- 3) For any event A , $0 \leq P(A) \leq 1$
- 4) The sum of the probs of all simple events must equal 1

In fact,

- 5) For any event, the sum of the probs of each simple event must equal the prob of the event

Suppose our class is distributed this way:

| | From Mass | Not From Mass | |
|-------|-----------|---------------|----|
| Fresh | 3 | 5 | 8 |
| Soph | 4 | 10 | 14 |
| Jun | 8 | 8 | 16 |
| Sen | 5 | 7 | 12 |
| | 20 | 30 | 50 |

Assuming individuals are selected at random, the probabilities for different categories will equal the proportions of those categories.

Historical proportion = Future probability

So: $P(\text{from Mass})$

$P(\text{Senior})$

$P(\text{Fresh or Soph})$

- Suppose an unfair coin has heads twice as likely as tails. Find $P(H)$ & $P(T)$.
- Suppose a die has 1,2,3,4,5 all equally likely, but 6 as likely as all the others combined. Find each prob.
- Suppose I have a sample space with simple events A, B, C & D. Suppose A is twice as likely as B, and B is twice as likely as C, and C is twice as likely as D. Find the prob of each.

Now suppose we have equally likely simple events (or equally likely outcomes).

- A fair die has six sides. What is the prob of each?

- What if a sample space has N equally likely outcomes?

$$P(\text{each simple event}) = ?$$

- What if we choose an event of size S from a sample space with N equally likely outcomes? What will the prob of that event be?

Blood Types and Rh factor in the United States

(Contingency table)

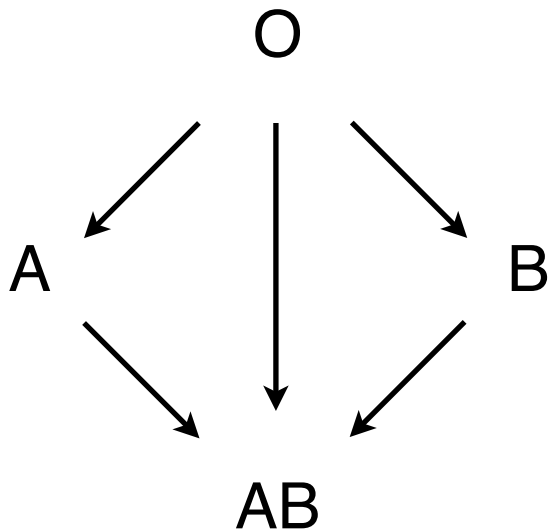
| | A | B | AB | O |
|-----|----|---|----|----|
| Rh+ | 36 | 8 | 3 | 37 |
| Rh- | 6 | 2 | 1 | 7 |

These are the percentages of each type of blood in the U.S.

Rh+ means your blood contains a certain substance

Rh- means your blood is missing that substance

If you are Rh-, you cannot tolerate/receive blood that is Rh+.



Some state lotteries are the type where you select three digits, each from 0-9.

Q1: How many different strings of this type are there?

Q2: Are any of those strings more or less likely than the others?

Q3: So how much probability do we assign to each individual number?

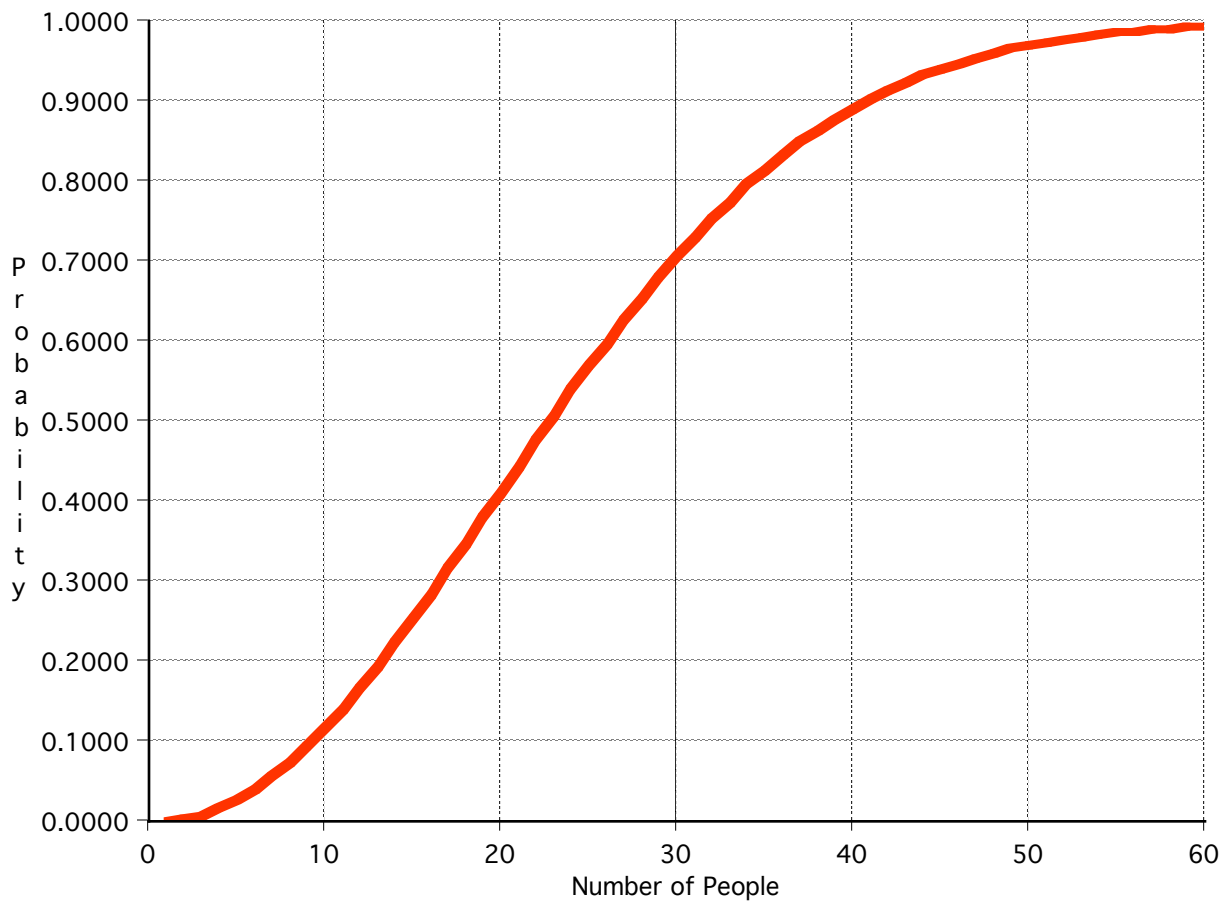
Q4: What is the probability that on 6/12 the number is 612?

Q5: What is the probability that on 6/13 the number is 612?

Q6: What is the probability that on 6/12 & 6/13 the two numbers match?

Birthday Problem

$P(\text{there are matches when there are } n \text{ people})$



| n | Prob |
|----|--------|
| 5 | 0.0271 |
| 10 | 0.1170 |
| 15 | 0.2529 |
| 20 | 0.4114 |
| 25 | 0.5687 |
| 40 | 0.8912 |
| 50 | 0.9704 |