# Trig 4.1: Trig Identities

We remember that the word "Trigonometry" comes from the Greek, meaning "triangle measurement." So far our study has dealt with things involving circles and triangles and other things like that.

Now we are ready to move beyond that into Algebraic Trig. That is, we want to start applying ideas of algebra to trigonometric expressions.

For example, let's solve the equation $\sin(x) + 1 = 0$
That is, let's find all values of x that make the equation true.
We can rearrange the equation to say $\sin(x) = -1$, so our job is to find all x values where $\sin(x)=-1$.
There's more than one way to think about this, but the easiest is to think with a unit circle. On the unit circle, sin(x) is the y-value, so we need to find all angles x that cause the y-value on the unit circle to equal -1. I hope it's obvious that this would be at the very bottom of the unit circle, which is when the angle is $3\pi/2$. That is: $x = 3\pi/2$. But we could go all the way around the unit circle another time, either clockwise or counterclockwise, and multiple times. So we could also have $7\pi/2$, $11\pi/2$, $15\pi/2$, ... , as well as $-\pi/2$, $-5\pi/2$, $-9\pi/2$, ... .
We will come back to this kind of problem later.

Another example is not quite so obvious at first: $\sin^2(x) - \cos^2(x) + 1 = 0$ .
Who knows what to do here? But if I told you that, using some of the properties we know, we could reduce that equation to $\sin(x) = 0$, then we would be back to a similar problem to the one above.
Again, we will spend more time with this type of problem later.

As you can see from that last problem, solving these kinds of equations will sometimes be simpler if we have some tools (properties) that we can use. So the rest of this lesson will be a reminder of some of the tools, and an introduction to others.

Before we can manipulate, we need to understand the difference between two types of equations:

An identity: An equation that is always true
An open sentence: An equation that is sometimes true, sometimes false

So when we talk about "solving equations" we are meaning finding the values that make an open sentence true.

But we will use identities to manipulate those equations.

In class we were going to run through the following list of identities, but today I will just list them out. I hope you recognize all of them. You need not have them all memorized, but you need to remember they exist (so you know to get them when you need them), and the more of them you remember, the easier your life will be (at least your mathematical life).
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## Library of Elementary Identities

**Reciprocal**

R1) $\sin(x) = \dfrac{1}{\csc(x)}$

R2) $\cos(x) = \dfrac{1}{\sec(x)}$

R3) $\tan(x) = \dfrac{1}{\cot(x)}$

**Pythagorean**

P1) $\sin^2 x + \cos^2 x = 1$

P2) $\tan^2 x + 1 = \sec^2 x$

P3) $\cot^2 x + 1 = \csc^2 x$

**Quotient**

Q1) $\tan(x) = \dfrac{\sin(x)}{\cos(x)}$

Q2) $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$

**Even/Odd**

EO1) $\sin(-x) = -\sin(x)$

EO2) $\cos(-x) = \cos(x)$

EO3) $\tan(-x) = -\tan(x)$

EO4) $\cot(-x) = -\cot(x)$

EO5) $\sec(-x) = \sec(x)$

EO6) $\csc(-x) = -\csc(x)$

**Cofunction/Complements**

CC1) $\sin(\frac{\pi}{2} - x) = \cos(x)$

CC2) $\cos(\frac{\pi}{2} - x) = \sin(x)$

CC3) $\tan(\frac{\pi}{2} - x) = \cot(x)$

CC4) $\cot(\frac{\pi}{2} - x) = \tan(x)$

CC5) $\sec(\frac{\pi}{2} - x) = \csc(x)$

CC6) $\csc(\frac{\pi}{2} - x) = \sec(x)$

We're going to use some of those identities to help us manipulate and simplify some trig expressions. And I've got some strategies to help you decide just what to do.

Strategy 1: Get a common argument

$$\frac{\sin(-x)}{\cos(x)}$$

The argument is the part inside the parentheses of the trig function, so it's -x for sine and x for cosine this time. We will want them to be the same. Notice that the Even/Odd Identities deal with + & - signs inside the argument, so we switch sin(-x), and then we use the first quotient property, to get:

$$\frac{\sin(-x)}{\cos(x)} = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$$

X1: You try this one. (Answer on the next page.)

Simplify cos(x) · cos(-x) - sin(x) · sin(-x)

Strategy 2: Get a common trig function

$$\frac{1+\tan(y)}{1+\cot(y)}$$

If we can easily change one trig function into the other, that might be helpful. This time, we can easily turn cot(y) into tan(y) because cot(y) = 1/tan(y). So:

$$\frac{1+\tan(y)}{1+\cot(y)} = \frac{1+\tan(y)}{1+\frac{1}{\tan(y)}} = \frac{(1+\tan(y))\tan(y)}{\left(1+\frac{1}{\tan(y)}\right)\tan(y)} = \frac{(1+\tan(y))\tan(y)}{(\tan(y)+1)} = \tan(y)$$

X2: You try this one. (Answer on the next page.)

$$\frac{1+\cos(y)}{1+\sec(y)}$$

Answer to X1: $\cos(x) \cdot \cos(-x) - \sin(x) \cdot \sin(-x)$
$= \cos(x) \cdot \cos(x) - \sin(x) \cdot -\sin(x) = \cos^2(x)+\sin^2(x) = 1$

Answer to X2:

$$\frac{1+\cos(y)}{1+\sec(y)} = \frac{1+\cos(y)}{1+\frac{1}{\cos(y)}} = \frac{(1+\cos(y))\cos(y)}{\left(1+\frac{1}{\cos(y)}\right)\cos(y)} = \frac{(1+\cos(y))\cos(y)}{(\cos(y)+1)} = \cos(y)$$

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Strategy 3: Convert to sine & cosine

$\tan(A)\ \csc(A)$

It is easy to change our trig functions into sine and/or cosine using reciprocals or the quotients. Then we can try to cancel out those sines or cosines, or perhaps turn sine or cosine into the other.

$$\tan(A)\ \csc(A) = \frac{\sin(A)}{\cos(A)}\frac{1}{\sin(A)} = \frac{1}{\cos(A)} = \sec(A)$$

Strategy 4: Try to factor the expression

$\tan^2 x - \tan^2 x \sin^2 x$

If you can factor it, you might notice something you can substitute in, and then move to another strategy.

$$\tan^2 x - \tan^2 x \sin^2 x = \tan^2 x(1-\sin^2 x) = \tan^2 x \cos^2 x = \frac{\sin^2 x}{\cos^2 x}\frac{\cos^2 x}{1} = \sin^2 x$$

Strategy 5: Get a common denominator

$$\frac{1}{1+\cos(x)}+\frac{1}{1-\cos(x)}$$

If your expression has a denominator or two in it, you might try to get a common denominator to help things simplify. On our example, the common denominator would be $(1+\cos(x))(1-\cos(x))$, so we multiply top & bottom of each piece by the missing denominator piece.

$$\frac{1}{1+\cos(x)}+\frac{1}{1-\cos(x)} = \frac{1}{(1+\cos(x))}\frac{1-\cos(x)}{(1-\cos(x))}+\frac{1}{(1-\cos(x))}\frac{1+\cos(x)}{(1+\cos(x))}$$

$$= \frac{1-\cos(x)+1+\cos(x)}{(1+\cos(x))(1-\cos(x))} = \frac{2}{1-\cos^2(x)} = \frac{2}{\sin^2(x)} = 2\csc^2(x)$$

Some more for you to try. Answers are on the next page.

X3. Simplify $\frac{\sin(x)\sec(x)}{\tan(x)}$

X4. Simplify $\frac{\cos^2(x)}{1-\sin(x)}$

X5. Simplify $\frac{2+\tan^2(x)}{\sec^2(x)}-1$

Answers from the previous page:

X3. $\frac{\sin(x)\sec(x)}{\tan(x)} = \frac{\sin(x)\frac{1}{\cos(x)}}{\tan(x)} = \frac{\frac{\sin(x)}{\cos(x)}}{\tan(x)} = \frac{\tan(x)}{\tan(x)} = 1$

X4. $\frac{\cos^2(x)}{1-\sin(x)} = \frac{1-\sin^2(x)}{1-\sin(x)} = \frac{(1-\sin(x))(1+\sin(x))}{1-\sin(x)} = 1 + \sin(x)$

X5. $\frac{2+\tan^2(x)}{\sec^2(x)}-1 = \frac{2+\tan^2(x)}{\sec^2(x)}-\frac{\sec^2(x)}{\sec^2(x)} = \frac{2+\tan^2(x)-\sec^2(x)}{\sec^2(x)} = \frac{2+\sec^2(x)-1-\sec^2(x)}{\sec^2(x)}$

$= \frac{1}{\sec^2(x)} = \cos^2(x)$

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One last thing we need to do, which is actually a simplification of what we are doing above. That is when we are requested to “Verify an Identity.” To do this, we are taking an equation that we know is true and proving that it is true, simply by manipulating one side to look like the other side.

I call this easier to what is above because in this case we know what the final answer is supposed to look like. Our job is just to get there.
NOTE: When you do this, you can start on either side, but then you manipulate until your final answer is the other side.

Example: Verify $\frac{\sin(t)\sec(t)}{\tan(t)+\cot(t)}=\sin^2 t$

We work on the left, changing it to the right.

$$\frac{\sin(t)\sec(t)}{\tan(t)+\cot(t)} = \frac{\sin(t)\frac{1}{\cos(t)}}{\frac{\sin(t)}{\cos(t)}+\frac{\cos(t)}{\sin(t)}} = \frac{\sin(t)\frac{1}{\cos(t)}}{\frac{\sin(t)}{\cos(t)}+\frac{\cos(t)}{\sin(t)}}\frac{\frac{\cos(t)\sin(t)}{1}}{\frac{\cos(t)\sin(t)}{1}} = \frac{\sin^2(t)}{\sin^2(t)+\cos^2(t)}$$

$$= \frac{\sin^2(t)}{1} = \sin^2(t)$$

Your homework for this section

From your book, do Section 4.1 # 9,15,29,33,45,49,57,73
Since you will always know the answers to these (either because the answers are in the back or because you are verifying identities), you do not need to submit your work to me. You are simply going to email me to tell me that you did the work. I WILL TRUST YOU!

Eventually I will post solutions to these problems, but if you want feedback on your work, you can scan or photograph some or all of your work and send it to me. I will try to give you that feedback. But this is not required. Only if you think it will help you.