Trig 4.2-3: More about Identities

(Because you loved them so much last time!)

Today we look at a few more trig identities that usually are needed when you mix angles together or take angles apart.

As I said last time, you need not have these all memorized, but you need to remember they exist (so you know to get them when you need them), and the more of them you remember, the easier your mathematical life will be.

And one more comment there: You will really impress your Calculus 1 instructor next semester when he/she asks “Does anyone remember the double-angle formula for sine?,” or some other similar question.

More Trig Identities

The sum of angles formulas: (You are adding angles A & B together)

$\sin(A+B) = \sin A \cos B + \sin B \cos A$
$\cos(A+B) = \cos A \cos B - \sin A \sin B$

The difference of angles formulas: You are subtracting angle B from angle A)

$\sin(A-B) = \sin A \cos B - \sin B \cos A$
$\cos(A-B) = \cos A \cos B + \sin A \sin B$

The double-angle formulas: (You are doublling angle A)

$\sin(2A) = 2 \sin A \cos A$
$\cos(2A) = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

There are three versions of this last one, because of the Pythagorian Identities.

The half-angle formulas: (You are dividing the angle A in half)

$$\sin(A/2) = \pm\sqrt{\frac{1-\cos(A)}{2}}$$

$$\cos(A/2) = \pm\sqrt{\frac{1+\cos(A)}{2}}$$

These last two have ± in front, because we cannot tell in general which quadrant we are in until we know what A is. Once we know A, and then A/2, we can choose + or -.

Before we do some examples, let's remember that:

$\sin(30°)=1/2$ $\cos(30°)=\sqrt{3}/2$
$\sin(45°)=\sqrt{2}/2$ $\cos(45°)=\sqrt{2}/2$
$\sin(60°)=\sqrt{3}/2$ $\cos(60°)=1/2$

Some Examples

We are doing ALL of these exactly, so using the formulas, not a calculator.

a) Find cos 15°

Notice that 15° = 45°-30°, so we can use the cosine difference of angles formula

$$\cos(45°-30°) = \cos(45°)\cos(30°)-\sin(45°)\sin(30°)$$
$$= \sqrt{2}/2 \times \sqrt{3}/2 + \sqrt{2}/2 \times 1/2$$
$$= (\sqrt{6} + \sqrt{2})/4$$

b) sin 22.5°

Notice that 22.5° is half of 45°, so we use the sine half-angle formula, using A=45° and A/2=22.5°. Since we know that 22.5° is in the first quadrant, we know that sine will be positive, so we use the + part of the formula.

$$\sin 22.5° = +\sqrt{\frac{1-\cos(45^o)}{2}} = +\sqrt{\frac{1-\sqrt{2}/2}{2}}$$

We can leave the answer right there, but it does simplify just a little to

$$\frac{\sqrt{2-\sqrt{2}}}{2}$$

c) Verify the identity $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$

We start with the left and manipulate to the right.

$\sin(A+B)\sin(A-B) = (\sin A \cos B + \sin B \cos A)(\sin A \cos B - \sin B \cos A)$

$= \sin^2 A \cos^2 B - \sin A \cos B \sin B \cos A + \sin B \cos A \sin A \cos B - \sin^2 B \cos^2 A$

$= \sin^2 A \cos^2 B - \sin^2 B \cos^2 A$

$= \sin^2 A (1-\sin^2 B) - \sin^2 B (1-\sin^2 A)$

$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 B \sin^2 A$

$= \sin^2 A - \sin^2 B$

You try these. Answers on the next page.

X1. If cos x = 3/5, find cos2x, and then cos4x.

X2. Verify this identity: $$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \sec(x)$$

X3. Verify this identity: $$\frac{\sin(A+B)}{\sin(A-B)} = \frac{\tan(A)+\tan(B)}{\tan(A)-\tan(B)}$$

Answer to X1. If cos x = 3/5, find cos2x, and then cos4x.
We first use the second version of the double-angle for cosine formula.

$$\cos(2x) = 2\cos^2 x - 1 = 2(3/5)^2 - 1 = 2(9/25) - 1 = 18/25 - 1 = -7/25.$$

Now we use the same double angle formula.

$$\cos(4x) = 2\cos^2 2x - 1 = 2(-7/25)^2 - 1 = 2(49/625) - 1 = 98/625 - 1 = -527/625.$$

Answer to X2. Verify this identity: $\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \sec(x)$

$$\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \frac{2\sin(x)\cos(x)}{\sin(x)} - \frac{2\cos^2(x)-1}{\cos(x)} = 2\cos(x) - 2\cos(x) + \frac{1}{\cos(x)}$$

$$= \frac{1}{\cos(x)} = \sec(x)$$

Answer to X3. Verify this identity: $\frac{\sin(A+B)}{\sin(A-B)} = \frac{\tan(A)+\tan(B)}{\tan(A)-\tan(B)}$

$$\frac{\sin(A+B)}{\sin(A-B)} = \frac{\sin A\cos B + \sin B\cos A}{\sin A\cos B - \sin B\cos A} = \frac{\sin A\cos B + \sin B\cos A}{\sin A\cos B - \sin B\cos A} x \frac{1/\cos A\cos B}{1/\cos A\cos B}$$

$$= \frac{\sin A\cos B/\cos A\cos B + \sin B\cos A/\cos A\cos B}{\sin A\cos B/\cos A\cos B - \sin B\cos A/\cos A\cos B} = \frac{\sin A/\cos A + \sin B/\cos B}{\sin A/\cos A - \sin B/\cos B} = \frac{\tan A + \tan B}{\tan A - \tan B}$$

Your homework for these sections

Sec. 4.2: # 1,3,11,13,27,35
Sec. 4.3: # 1,17,23,29,53,58

As before, since you will always know the answers to these (either because the answers are in the back or because you are verifying identities), you do not need to submit your work to me. You are simply going to email me to tell me that you did the work. I WILL TRUST YOU!

Eventually I will post solutions to these problems, but if you want feedback on your work, you can scan or photograph some or all of your work and send it to me. I will try to give you that feedback. But this is not required. Only if you think it will help you.