Trig 4.5b: Solving Trig Equations, Part 2

We have two things to add to our skills of solving equations: What if the argument is not just x? and what if our equation is not linear?

The <u>argument of a trig function</u> is the part inside the function. So the argument for $\sin(4x)$ is $4x$, and the argument for $\cos(x^2)$ is x^2, and the argument for $\tan(3x+1)$ is $3x+1$.

If we need to solve a trig equation, and the argument for our trig function is something other than just x, then we will have one last thing to do at the end of our problem.

Example 1. Solve $2\sin(5x) - 1 = 0$.
This is linear in $\sin(5x)$, so we get $\sin(5x) = 1/2$.
Sine equals 1/2 twice per cycle, in the 1st & 2nd quadrant, and 1/2 comes from a 30-60-90 triangle, so we get a 1st quadrant angle of $\pi/6$, and then a 2nd quadrant angle of $5\pi/6$. So our solution (this far) is:

$$5x = \pi/6 + 2\pi n$$

and $$5x = 5\pi/6 + 2\pi n$$

But we need to isolate x, so we divide through by 5, to get

$$x = \pi/30 + 2\pi n/5$$

and $$x = \pi/6 + 2\pi n/5$$

BE AWARE: Your book does this in a different way, one that is much more difficult (I think). I highly recommend you follow my path here, not theirs.

Example 2. Solve $\tan^2(8x) - 1 = 0$
Linear in $\tan^2(8x)$, so we get $\tan^2(8x) = 1 \Rightarrow \tan(8x) = \pm 1$
First we solve $\tan(8x) = 1$. Tangent equals 1 once per cycle, in the 1st quadrant, so we get, and 1 comes from a 45-45-90 triangle, so we get a 1st quadrant angle of $\pi/4$. Tangent equals -1 once per cycle, in the 4th (or 2nd, doesn't matter) quadrant, so we get a 4th quadrant angle of $-\pi/4$. Thus

$$8x = \pi/4 + \pi n$$

and $$8x = -\pi/4 + \pi n,$$

So

$$x = \pi/32 + \pi n/8$$

and $$x = -\pi/32 + \pi n/8.$$

Now, <u>what if our equation is not linear?</u> To solve a linear equation, you move the variables to one side and the non-variables to the other side.
To solve a non-linear equation, you move everything to one side, and then you try to factor that side. Because if the whole equation equals 0 (because moved everything to one side), and if you can factor the expression, then the individual factors must equal zero (since two things can only multiply to zero if one or the other of the things is zero).

Example 3: Solve $2 \sin^2 x - \sin x = 1$.
This is not linear, so we move everything to one side by subtracting 1 from both sides. $2 \sin^2 x - \sin x -1 = 0$
Now we try to factor. This is a quadratic (2nd power) problem, so you may remember working with them in a previous class. Basically we are trying to Un-FOIL it. If it helps you to temporarily replace the sin(x) with something else, like a y, then do that. You would get $2y^2 - y - 1 = 0$
I'll continue with the given expression.
We need to factor the equation into a form like this: $(a + b)(c + d) = 0$.
We need the leading pieces (a & c) to multiply to $2\sin^2 x$, so we split it into 2sin(x) and sin(x). That gives $(2\sin x + b)(\sin x + d)$
Next we need the b & d to multiply to -1, so 1 & -1, but we can't tell which is which until we try them. So we go to trial and error. If we let -1 be b and 1 be d, we have $(2\sin x - 1)(\sin x + 1)$.
Now we FOIL back out to see if we got the right thing:

$2\sin^2 x + 2\sin x - 1\sin x - 1$, which is $2\sin^2 x + \sin x -1$, which is not the right thing. So we try the other way.

We have $(2\sin x + 1)(\sin x - 1)$.
Now we FOIL back out to see if we got the right thing:

$2\sin^2 x - 2\sin x + 1\sin x - 1$, which is $2\sin^2 x - \sin x -1$, which IS the right thing. So we try the other way.

So our equation is now $(2\sin x + 1)(\sin x - 1) = 0$.
Now we go to the next phase: We break the problem into two smaller problems.

$(2\sin x + 1) = 0$ and $(\sin x - 1) = 0$

and we proceed to solve them.

$2\sin x + 1 = 0 \Rightarrow \sin(x) = -1/2 \Rightarrow x= 7\pi/6$ & $11\pi/6$.
$\sin x - 1 = 0 \Rightarrow \sin(x) = 1 \Rightarrow x = \pi/2$

So our solutions are:

$x= 7\pi/6 + 2\pi n$
$x = 11\pi/6. + 2\pi n$
$x = \pi/2 + 2\pi n$

Example 4. Solve sin(2x) = 2tan(2x)
Not linear, so $\sin(2x) - 2\tan(2x) = 0$
Let's plug in the quotient property for tangent, so

$$\sin(2x) - 2\frac{\sin(2x)}{\cos(2x)} = 0$$

We want to factor this, so we need to put those pieces together with a common denominator.

$$\frac{\sin(2x)\cos(2x)}{\cos(2x)} - 2\frac{\sin(2x)}{\cos(2x)} = 0$$

$$\Rightarrow \quad \frac{\sin(2x)\cos(2x) - 2\sin(2x)}{\cos(2x)} = 0$$

A fraction equals zero when the numerator equals zero, not the denominator. So now we look just at the numerator.

$$\sin(2x)\cos(2x) - 2\sin(2x) = 0$$

Factor

$$\sin(2x)[\cos(2x) - 2] = 0$$

Now set the factors to zero and solve.

$\sin(2x) = 0 \quad \Rightarrow \quad 2x = 0$ and $2x = \pi$ in the first cycle
$\cos(2x) - 2 = 0 \quad \Rightarrow \quad \cos(2x) = 2$, which never happens.

So $2x = 0 + 2\pi n$
and $2x = \pi + 2\pi n$
And finally
$x = 0 + \pi n$
and $x = \pi/2 + \pi n$

Example 5. Solve $(2\cos(x) + \sqrt{3})(2\sin(x) - 1) = 0$
Not linear, but everything is already moved to one side and factored. So:
$(2\cos(x) + \sqrt{3}) = 0 \quad \Rightarrow \quad \cos(x) = -\sqrt{3}/2$
Reference angle is $\pi/6$, but we need quadrants 2 & 3
$x = 5\pi/6$ and $7\pi/6$.

$(2\sin(x) - 1) = 0 \quad \Rightarrow \quad \sin(x) = 1/2$
Reference angle is $\pi/6$, but we need quadrants 1 & 2
$x = \pi/6$ and $5\pi/6$
Final answer:
$x = \pi/6 + 2\pi n$
$x = 5\pi/6 + 2\pi n$
$x = 7\pi/6 + 2\pi n$

<u>Your homework for this lesson:</u>

Section 4.5: Please do # 11, 16, 22, 23, 26, 32, then email your answers to me.