Vectors & Trigonometry, Part 1

This workbook is intended to be a supplement to Section 5.5 in your text. Make sure you read your textbook as well.

Today we start looking at a new type of mathematical object: a vector.

Numbers are mathematical objects. If we use numbers to tell us about a physical situation, each number can tell us one thing about that situation.

- We can say a car is going 65 mph, but the number "65" tells us the speed of the car and nothing else. What direction we're traveling, if we're speeding up or slowing down, how old the driver is, etc.

Vectors are mathematical objects that hold two pieces of information at once. Those two pieces are usually referred to as the direction and the magnitude.

Some examples of situations where vectors fit nicely are:

Example 1) Suppose an airplane is flying toward the northeast at 200 mph. At the same time the wind is blowing directly out of the north at 50 mph. As a result, the airplane actually travels (with respect to the ground) at about 168 mph in a direction about 33 degrees north of east. The vectors that are in play here are:

The airplane vector itself (w/o wind): direction = 45 deg, mag = 200
The wind vector: direction = -90 deg, mag = 50
The resultant vector: direction = 33 degrees, mag = 168

These vectors are all "velocity vectors."

Example 2) Suppose a river is running from north to south with a current speed of 10 mph. Suppose a boat aims directly across the river (i.e. directly east), and the boat has a motor that could move the boat at 18 mph in still water. The vectors in play here are: (You describe them)

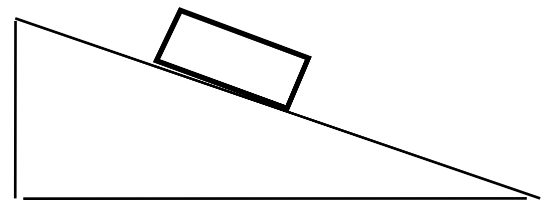
The boat:

The river:

The resultant:

These are again "velocity vectors."

Example 3) A block of wood is sitting on a slope, as shown to the right. The slope has friction which prevents the wood from sliding down the hill. Identify the “force vectors” in effect here.



Geometric descriptions of vectors

A good way to think of a vector (in fact, this is what your text uses as a definition) is as a directed line segment: that is, as a line segment pointing in the direction of the vector of interest, where the length of the line segment corresponds to the magnitude of the vector.

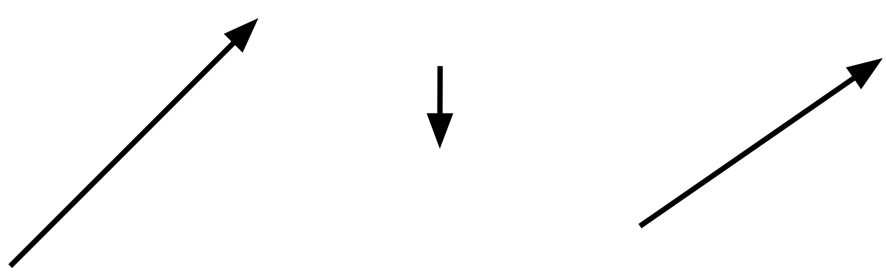
Example 1, again) Our vectors are:

The airplane vector: direction = 45 deg, mag = 200

The wind vector: direction = -90 deg, mag = 50

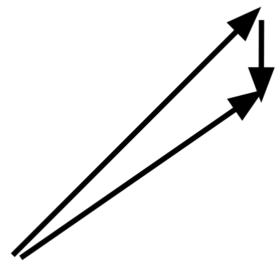
The resultant vector: direction = 33 degrees, mag = 168

We sketch these below:



Notice that the airplane vector (on the left) is the longest because it has the greatest magnitude. The resultant vector (on the right) is not quite as long as the airplane vector, and not quite as steep.

Example 1, continued) If we rearrange those vectors, you might notice something interesting:



Observe: If we put the two original vectors (airplane and wind) end-to-end, the resultant vector will fill in the gap (i.e. run from the starting point of the first to the ending point of the second vector).

This is part of what makes working with vectors so valuable: the simple task of placing the vectors end-to-end identifies for you what happens when you combine the effects of the vectors.

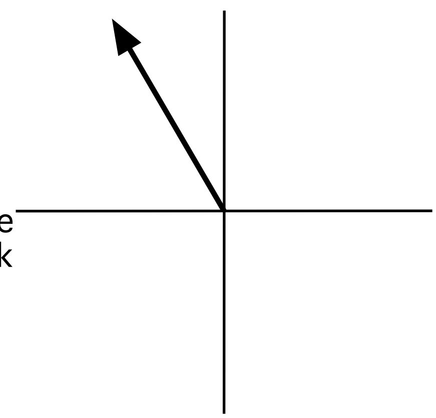
Definition: The task of placing vectors end-to-end and identifying the resultant vector is called "adding vectors" (geometric version), and the resultant vector is also known as the "sum."

Example 2, again) The river is running from north to south with a current speed of 10 mph, and the boat is aimed directly east with a speed of 18 mph. Geometrically add the two vectors together to identify the resultant vector.

Example 2, still) Let's suppose we actually want to end up straight across the river. Our previous plan won't work because we will end up downstream too far. See if you can geometrically rearrange our problem so we end up straight across the river (i.e. the resultant points straight to the east).
Hint: Considering the boat vector and the river vector, you only have control over one of those.

Identifying the direction

There are several different schemes for identifying the direction a vector points, all of them working equally well. It usually does not matter which scheme you use, but once you choose a scheme for a problem, you should stick with it for the remainder of the problem. Also, sometimes the choice is made for you, so you need to know how to work with each. We will describe the vector to the right using each of the schemes. That vector is supposedly pointing about 30 degrees around from the y-axis.



Scheme 1: Traditional trigonometry approach. Simply identify the size of the standard position angle that would be needed to have your vector on the terminal side.

In the above case, the angle would be $\theta = 90° + 30° = 120°$

Scheme 2: Heading. In this case you consider straight north to be 0°, and you circle clockwise around until you reach your vector.

In the example, the vector is 30° short of 360°, so the heading is 330°.

Scheme 3: Common approach. You identify the direction by how far from one of the four main directions (NSEW) it is.

In the example, the closest direction is north, and you would go 30° toward the west from that direction, so we call it 30° West of North. (Sometimes it's phrased as "North 30° West.")

Homework: Geometric descriptions of vectors

Your job is to solve the following problems geometrically. That is, you should draw directed line segments that represent the vectors of the problem, and use those directed line segments to answer the questions of the problems.

1. A river flows due south at 3 mi/h. A swimmer attempting to cross the river heads due east swimming at 2 mi/h relative to the water. Draw those vectors, and the true velocity vector of the swimmer. (Reminder: The true vector would be the resultant vector when you add the other two vectors together.) Then find the magnitude and direction of that true velocity vector.

2. A pilot heads his jet due east. The jet has a speed of 425 mi/h relative to the air. The wind is blowing due north with a speed of 40 mi/h. Draw those vectors, and the true velocity vector of the jet. Then find the magnitude and direction of that true velocity vector.

3. A woman walks due west on the deck of an ocean liner at 2 mi/h. The ocean liner is moving due north at a speed of 25 mi/h. Draw those vectors, and the true velocity vector of the woman. Then find the magnitude and direction of that true velocity vector.

4. A migrating salmon heads in the direction N 45° E, swimming at 5 mi/h relative to the water. The prevailing ocean currents flow due east at 3 mi/h. Draw those vectors, and the true velocity vector of the fish. Then find the magnitude and direction of that true velocity vector. (You will have to use the Law of Cosines on this one.)