Vectors & Trigonometry, Part 2

This workbook is intended to be a supplement to Section 5.5 in your text. Make sure you read your textbook as well.

Today we continue our look at a new type of mathematical object: a vector.

Algebraic descriptions of vectors

Since directed line segments work so well for describing vectors, our algebraic approach will essentially be to identify the important features of the directed line segments.

Notations:

- If a directed line segment (and thus a vector) has a displacement of "a" in the x direction and "b" in the y direction, we will denote the vector using those two numbers as follows: <a, b>.

- For convenience, we often denote a vector with a single name, either bolded or with an arrow over the top: A or $\vec{A}$

(The second notation gives a name to a vector, while the first notation reveals the important information about the vector.)

- We denote the magnitude of a vector A as follows: |A|. (There's a reason it looks like absolute value, and I'll be glad to explain that reason some time after class to anyone that's interested.

- There is not a common notation for the direction of a vector, just like there is not a common scheme for direction. I like to use "dir(A)" as a symbol for direction.

Example 1) Consider a vector A that has a displacement of 3 in the x direction and 4 in the y direction, and a vector B that has displacement of 4 in the x direction and 3 in the y direction. See if you can find the following:

$A = < \quad , \quad >$

$B = < \quad , \quad >$

$|A| =$

dir(A) =
(use the trig scheme)

$A + B = < \quad , \quad >$

Based upon what you saw in that example, see if you can identify formulas for the following:

If $A = <a_1, a_2>$ and $B = <b_1, b_2>$, then:

$|A| =$

dir(A) is found by

$A + B = \quad < \qquad , \qquad >$

$kA = < \qquad , \qquad >$ (k is a constant)

One more notation thing: There are two “basic vectors” that are often used:

i = <1,0> and j = <0,1>.

Every other vector could (and often is) written in terms of these vectors.

For example, the vector <3,4> = <3,0> + <0,4>
= 3<1,0> + 4<0,1>
= 3i + 4j

Example 2) Write the vector <5, -6> in i/j form

and write the vector -7i - 2j in bracket form.

Example 3) A boat has still-water speed of 4 mph. A river current is 3 mph. The boat aims perpendicular to the shore of the river. What is its actual speed, and its actual direction?

Example 4) Suppose we want the boat in the above example to travel straight across the river. What boat vector should we use, and what will be our actual speed?

Example 5) An airplane has a heading of 45° with an airspeed of 300 mph. The wind is heading 160° at 60 mph. Find the actual groundspeed and direction of the airplane.

Example 6) Suppose the airplane in the above example wants to actually travel with a resultant heading of 70°. What heading should it take, and how fast will it actually travel?