Exam date: Tuesday, February 4

- This exam consists of 6 problems (12 parts) and is worth 80 points.
- Partial credit will be given on most problems, but only for proper work which is shown.
- You may use a dedicated calculator on this exam, but not a “calculus doing” calculator. (That rules out a cell phone or iPod-type calculator.) You also will not be allowed to use any graphing features of your calculator.
- You may use a calculus book to help you with your integration. (Honor system: You must not look in the chapter on Diff Eqns during the exam.)

Topics

Classification
- Be able to classify DE's (ordinary/partial, order, linear, LH, NH)

Modeling
- Be able to model physical situations with DE's (i.e. take a word problem and turn it into a DE)

Existence & Uniqueness:
- Understand the importance of these theorems.
- Be able to test an IVP for existence & uniqueness of solutions. (The relevant theorem will be given.)

Solutions to DE's
- Be able to verify that a given function is a solution to a DE or IVP
- Be able to sketch and/or read a direction field for a DE. Also be able to sketch approximate solutions on the field.
- Be able to solve first-order DE's with our techniques from class

Other things
- Be able to construct a phase line for an autonomous DE, and know how to label the types of critical points.
- Be able to use a phase line to describe solutions to a DE with a given initial condition.
- Be able to construct a bifurcation chart, and identify where any bifurcations are.

Problems to practice (in addition to assigned homework problems)
Chap 1 Review Ex: 7-12, 15-18, 20-22
Chap 2 Review Ex: 3-7, 8acegkln, 9-18, 19, 22
Chap 3 Review Ex: 1-2, 4-5, 9

Diff Equations | Exam 1 | Name: _____________
Spring 2018 | 100 points

This exam consists of 8 problems (15 parts) and is worth 100 points. Show all of your work and final answers in the space provided. Extra paper is available, if needed. Partial credit will be given on most problems, but only for proper work which is shown.

1. (6 points) Euler's Law of Facebook Parties says: the number of people arriving at the party every minute is 8, while the number departing is equivalent to 10% of the number at the party. The party starts at 8:00 (time = 0) with 10 people present.

Define some variables and set up an IVP to model the change in the number of people at a Facebook Party.

(You do not need to solve the IVP.)

2. (6 points each) Consider this autonomous DE: $y' = y(2 - y/N)$.
(N is a positive constant.)

a) Draw the phase line for this DE (including the arrows!).
b) On your phase line, label each critical point as stable, unstable or semi-stable. (You may use the words repellers, attractors, and semi-stable, if you prefer.)
c) From what your phase line shows, describe the behavior of the solution we would get if we had an initial condition of $y(0) = N$.

4. (8 points each) Solve these ODE's using any of the techniques discussed in class.

c) $(2xy)\,dx + (x^2-1)\,dy = 0$
d) $y' + y/x = x^3$

7. (7 points) Consider the following IVP:

$$y' = xy^{2/3}$$
$$y(1) = -1$$

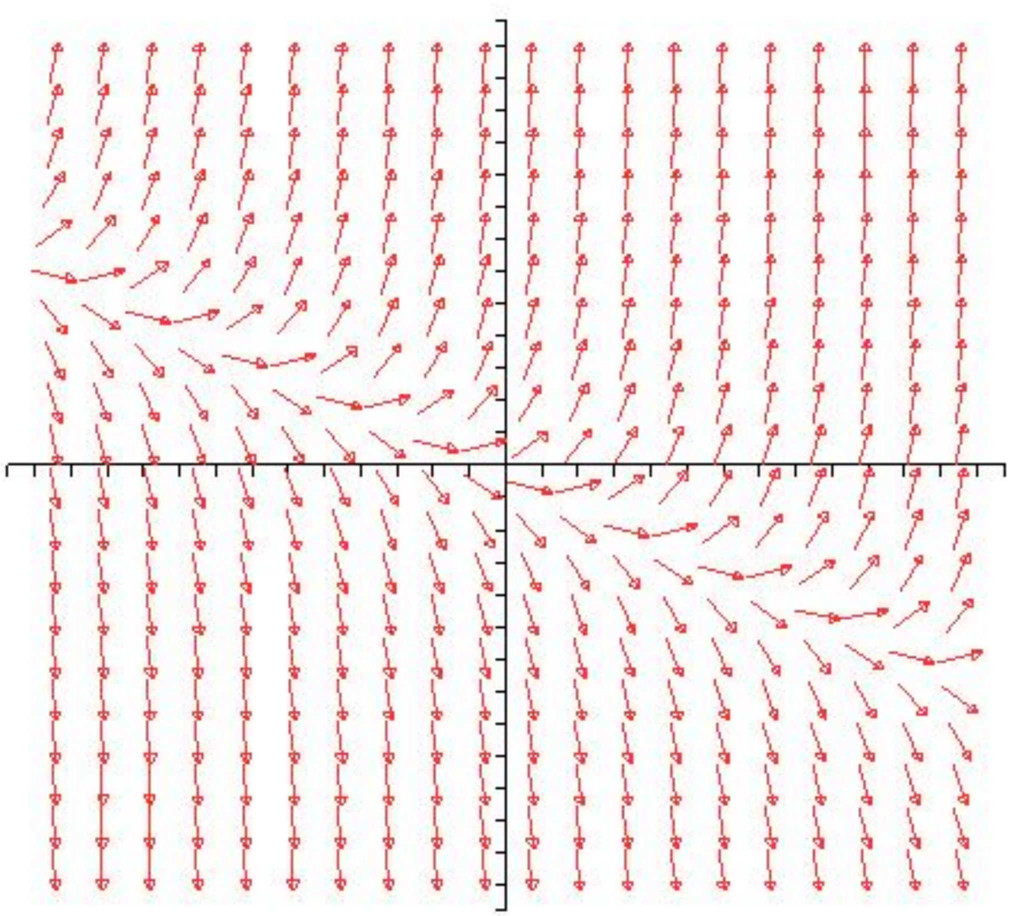
What does our existence and uniqueness theorem for first order differential equations tell us about this IVP? Briefly explain your answer.

8. (5 points) The differential equation $y' = y^2 + 1 - C$, where C is some parameter, is an example of a DE that leads to a bifurcation.

Do either part a or part b below. You may do both, and I will count the one that gives you the best score.

a) In your own words, explain what is meant by a bifurcation.
b) Sketch the bifurcation diagram for this DE.

3. (5 points each) Look at the direction field below for the differential eqn $y' = f(x,y)$.



a) Sketch an approximate solution curve that contains the point (0,-5).
b) There is one linear solution for the DE. That is, one solution such that if you start on the line you must stay on the line. Approximate where that solution must be.
c) Considering the linear solution you sketched in part (b), there is no solution to the DE that starts out above that linear solution and ends below that linear solution. Briefly explain how we know that is the case.

Diff Equations   Exam 1   Name: ______________________
Spring 2016   100 points

This exam consists of 8 problems (15 parts) and is worth 100 points. Show all of your work and final answers in the space provided. Extra paper is available, if needed. Partial credit will be given on most problems, but only for proper work which is shown.

1. (4 points each) Consider this autonomous DE: $y' = y^2(y^2-1)$.

a) Draw the phase line for this DE.
b) Using your phase line, describe what would happen if we had an initial condition of $y(0) = -2$.

2. (8 points each) Solve these ODE's using any of the techniques discussed in class.

a) $y' - ay = ce^{bx}$ (a, b & c are constants, and $a \neq b$)

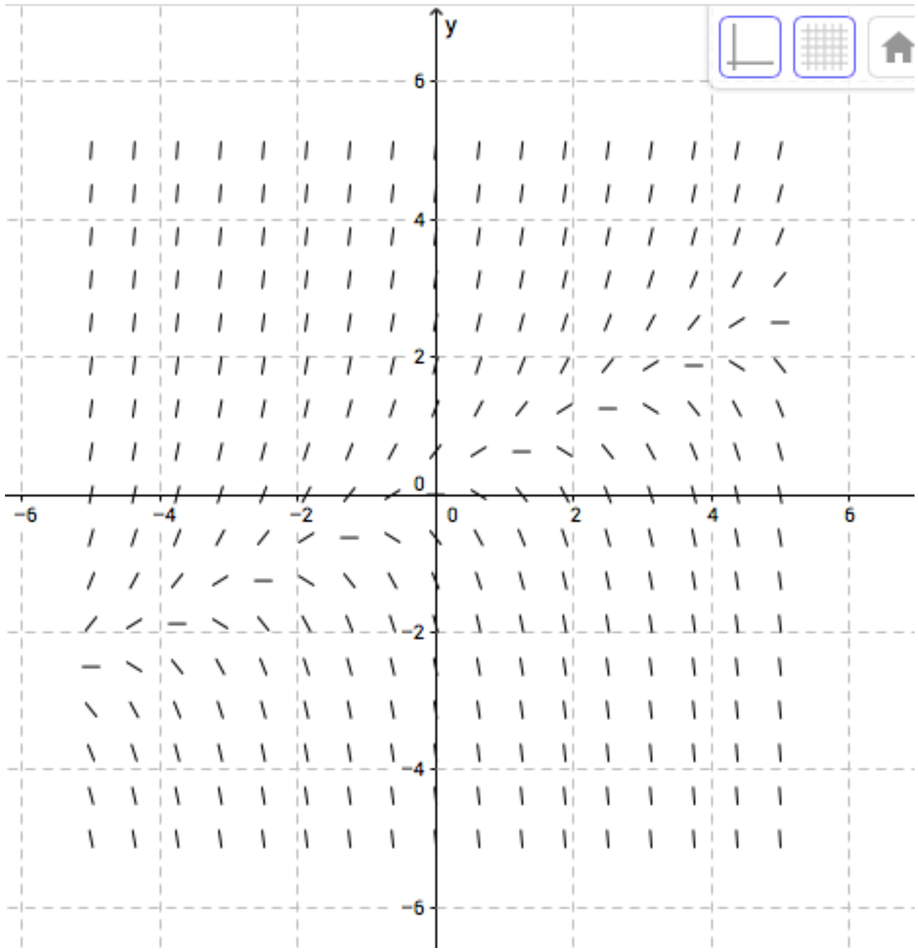
b) $(w^3 + wz^2 - z)\, dw + (z^3 + w^2z - w)\, dz = 0$

5. The brightness of a computer screen is measured on a scale of 0 to 1000, with 1000 being the very brightest. The brightness tends to fade as the screen ages. In fact, it is usually considered that the brightness changes (with respect to time) at a rate that is both proportional to the amount of time elapsed, but also proportional to its brightness. Let's suppose that we bought a computer screen in 2013, and its brightness then was 900. Now in 2016 the brightness is 800. (Think of the year 2013 as time t=0.)

a) (8 points) Find the differential equation that describes the change of brightness of the computer screen.

b) (8 points) Completely solve the DE from part (a). You should have no unknown constants when you are done.

8. (4 points each) Look at the direction field below for the differential equation $y' = 2y - x$.



a) There is one linear solution for the DE. That is, one solution such that if you start on the line you must stay on the line. Approximate where that solution must be. (Draw it right on the graph. There is no calculation to do.)

b) Sketch an approximate solution curve that contains the point (2, 0). (Draw it right on the graph above. If you mess up, I have extra copies of this page.)