

Diff Equations

Exam 2 Summary/Review

Exam date: Thursday, February 27

- This exam consists of 5 problems (7 parts) and is worth 60 points.
- Partial credit will be given on most problems, but only for proper work which is shown.
- You may use a dedicated calculator on this exam, but there's not a lot of calculating in this chapter.
- You may use a calculus book to help you with your integration. (Honor system: You must not look in the chapter on Diff Eqns during the exam.)
- You may use a single 3x5 note card (and you may write on both sides).

Topics

Solutions to DE's

Know what is meant by a fundamental set of solutions of an LH, a general solution for LH, a particular soln to NH, a complementary soln to NH, and a general soln to NH

Be able to solve LH & NH problems with constant coeffs

Other things

Understand and be able to work with differential operators D and L

Be able to show that functions are linearly independent, or demonstrate that they are linearly dependent.

Be able to prove:

$$L(\alpha y) = \alpha Ly$$

$$L(y_1 + y_2) = Ly_1 + Ly_2$$

If f_1, f_2, \dots, f_m are solns of LH, then so is $c_1 f_1 + c_2 f_2 + \dots + c_m f_m$

If v solves NH, and u solves LH, then $u+v$ solves NH.

Problems to practice (in addition to assigned homework problems)

Chap 4 Review Ex: 5-6, 7a, 9-14, 26a, 29-32

This exam consists of 8 problems (15 parts) and is worth 100 points. Show all of your work and final answers in the space provided. Extra paper is available, if needed. Partial credit will be given on most problems, but only for proper work which is shown.

1. (8 points each) Solve these ODE's using any of the techniques discussed in class.

a) $y'' - y' + 2y = 0$

b) $D(D-1)^2y = 0$

2. (8 points) Determine if the functions x , $x+1$, and $x+2$ are linearly independent or linearly dependent. (Be sure to show how you know.)

3. (9 points) Suppose we have the NH differential equation $Ly=G$. Suppose we know that the function f is one solution to the NH, and we also know that the function g is one solution to the corresponding LH problem $Ly=0$. Prove that the function $h = f+g$ is also a solution to the NH.

4. (15 points) Solve the following NH problem using either annihilators or variation of parameters. (Both work. It's your choice.)

$$y'' - y = e^x$$

1. (15 points) Solve the following NH problem using annihilators.
(If you can't solve it using annihilators, solve it another way for partial credit.)

$$y'' + y = xe^x$$

2. (15 points) Solve the following differential equation using variation of parameters.
(If you can't solve it using variation of parameters, solve it another way for partial credit.) (Don't forget the shortcut reminder on the help page.)

$$y'' + y = \sec^3(x)$$

3, continued. (8 points each) Solve these ODE's using any of the techniques discussed in class.

a) $y'' - 3y' + 2y = 0$

b) $y'' - 3y' + 2y = x^2$

e) $(D-1)(D+2)(D^2+1)y = 0$

4. (6 points) Determine if the functions 1, x, and 4-3x are linearly independent or linearly dependent. (Be sure to show how you know.)

5. (6 points) Determine if the functions e^{ax} and e^{bx} are linearly independent or linearly dependent. a & b are constants. (Be sure to show how you know.)

6. (8 points) Consider the following differential equations:

$$D(D-x)y = 0 \quad \text{and} \quad (D-x)Dy = 0 .$$

Expand each into the standard form (that is, using y' , not D) and observe that they are not the same. This will prove that the differential operators are not commutative.