Prob & Stats - Section 7.2: More Confidence Intervals

Last time we saw that a confidence interval for μ, under the right conditions, takes the following form

$$\bar{X} \pm z* \sigma/\sqrt{n}$$

$\bar{X}$ is the observed sample mean
z is the z-score that chops off the right amount of area
σ is the population standard deviation
n is the sample size.

The right conditions are:
σ is known
and either the pop is ND or $n \geq 30$.

If that second condition is not met, we can "Assume Pop ND" to be able to continue. If the first condition is not met, then we have a different problem. You might be inclined to substitute the good estimator s for σ, which is a fine idea. However, in the theory of statistics, it turns out that the statistic $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ has a ND(0,1), but the statistic $\dfrac{\bar{X}-\mu}{s/\sqrt{n}}$ does not. In fact, it has a different distribution called a t-distribution. A t-distribution is also basically bell-shaped, so if I showed you pictures of a z & a t side by side, you wouldn't be able to tell the difference. But the numbers (t-scores) you get are different from the z-scores.

In the case where we must use s instead of σ, we also must use t instead of z. Other than that, the confidence intervals work the same way. So we have these formulas:

Confidence intervals for μ

If σ is known, then $\bar{X} \pm z* \sigma/\sqrt{n}$

If σ is unknown, then $\bar{X} \pm t* s/\sqrt{n}$

How do we find t-scores? Well, using a t-table, of course! If you still have your z-table with you, then you also have a t-table on the back. If not, you can get one from the class website (on the bottom right), though I plan to attach one to this page, if I can remember.

Here's the very top of the t-table.

Critical t values					
	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
Degrees of Freedom	**Area in Two Tails**				
	0.01	0.02	0.05	0.10	0.200
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372

This table has some very important differences from the z-table. First, the z-table has the z-scores along the side and the top, and the areas inside the table. The t-table has the areas across the top, and the t-scores inside the table. The t-distribution also has one more parameter, called the degree(s) of freedom, (df) along the left edge. (Don't worry about why it has that name.) So, to find a t-score, we go to the row for the correct df (in a moment!), then go across to the column for the correct tail area.
Tail area: When we do a 95% CI, we are chopping out the middle 95% from the bell (or t) curve, and we are leaving 5% in the two tails. (For our purposes, confidence intervals are always two tail, though there is such a thing as a one tail CI.) So, for 95% CI, we use 0.05 area in the two tails.
df: For the kind of problem we are doing here, we calculate df = n-1.That is, one less than the sample size.

Examples: (Answers on the top of the next page)

a) Confidence level = 95%, n=6. Find t.

b) Confidence level = 99%, n=9. Find t.

c) Confidence level = 90%, n=11. Find t.

Answers from previous page:

a) t = 2.571 b) t = 3.355 c) t = 1.812

If the df you need happens to not be in the chart, you can either:

a) Round down to the next lower df.

b) unless it's much closer to the higher df, in which case you can use that higher df.

or c) You can interpolate.

Example: Suppose we are in the first column of that t-chart, and we have df=8.

7 gives us t = 3.499

9 gives us t = 3.250

Option (a) tells us to use df = 7, so our t-value would be 3.499.

Option (c) tells us, since 8 is halfway between 7 & 9, to use the t-value halfway between 3.499 and 3.250, or 3.375.

Example: Suppose we are still in the first column of the t-chart, and we have df=185.

Option (b) tells us to use df=200, so our t-value would be 2.601.

Option (c) tells us, since 185 is 85/100 of the way from 100 to 200, we go 85/100 of the way from 2.626 to 2.601. The difference is 0.025, and 85/100 of that is 0.025*0.85 = 0.021, so we go 0.021 down from 2.626, to get 2.605.

Example 1: 7. A sample was taken of 10 people who had just completed shoveling (manually) a tract of snow, and their maximum heart rates (beats per minute) were recorded. That sample yielded a mean of 175 with a standard deviation of 15. Find a 95% confidence interval of the mean maximum heart rate of all people manually shoveling snow.

Solution: My comments first. We want to find a 95% CI for μ. We observe that, while we were given a standard deviation, it was the std dev of the random sample, not the pop. So σ is unknown. That means we must use the t version of CI. Here's what I would write:

95% CI for μ with σ unknown

$\overline{X}$ = 175 s = 15 n = 10 t = 2.262

175 ± 2.262*15/sqrt(10) Assume pop ND

175 ± 10.7

[164.3, 185.7]

You try these. Answers on the next page

Example 2. In a sample of seven cars, each car was tested for nitrogen-oxide emissions (in grams per mile) and the following results were obtained:

0.06, 0.11, 0.16, 0.15, 0.14, 0.08, 0.15.

Construct a 95% CI of the mean NO-emissions for all cars. For your convenience, the sample yielded a mean of 0.1214 and a std dev of 0.0389.

Example 3. When people smoke, the nicotine they absorb is converted to cotinine, which can be measured. A sample of 40 smokers has a mean cotinine level of 172.5. Assuming that σ is known to be 119.5, find a 95% confidence interval estimate of the mean cotinine level of all smokers.

Example 2:

95% CI for μ with σ unknown

$\overline{X} = 0.1214$ s = 0.0389 n = 7 t = 3.707

$0.1214 \pm 3.707*0.0389/\sqrt{7}$ ASSUME POP ND

0.1214 ± 0.0545

[0.0669, 0.1759]

Example 3:

95% CI for μ with σ known

$\overline{X} = 172.5$ $\sigma = 119.5$ n = 40 z = 1.96

$172.5 \pm 1.96*119.5/\text{sqrt}(40)$

172.5 ± 37.03

[135.47, 209.53]

One more type of problem on the next page.

Quickly, I want to add one more CI formula, and that's for the parameter p. (Remember that p is the true proportion of success in some experiment, and we use the statistic $\hat{p}$ to estimate it. Following a similar path as we did in the last lesson, we end up with this CI formula:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

Example: I have a quarter, and I want to know what its true proportion of success (Heads) is. I toss the quarter 1000 times and observe 480 heads. Use a 95% CI to estimate the true proportion.

Solution: 95% CI for p

$\hat{p} = 480/1000 = 0.48$ $\hat{q} = 0.52$ $n = 1000$ $z = 1.96$

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} \Rightarrow 0.48 \pm 1.96 * \sqrt{\frac{0.48*0.52}{1000}} \Rightarrow 0.48 \pm 1.96*0.0158$$

$$\Rightarrow 0.48 \pm 0.031 \quad \Rightarrow \quad [0.449, 0.511]$$

Follow-up question: Does this interval support or refute the idea that this coin might be fair?

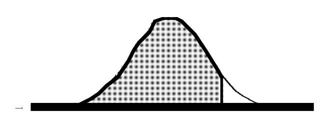
Answer: It supports fairness. All of the proportions inside the interval are reasonable true proportions. So 0.50, the fairness proportion, being inside the interval, is still a reasonable possibility.

Homework: From Section 7.2, #12, 13a, 19, 20

Calculate these, then either email just your answers to me, or scan/photograph your work and send it all to me.

A B C D E F G H I J K L M N O P Q R S T U V W

Standard Normal (z) Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0109
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1378
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3858
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6142
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8622
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9891
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Critical t values

Degrees of Freedom	Area in One Tail				
	0.005	0.01	0.025	0.05	0.10
	Area in Two Tails				
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1	63.657	31.821	12.706	6.314	3.078
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8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316
26	2.779	2.479	2.056	1.706	1.315
27	2.771	2.473	2.052	1.703	1.314
28	2.763	2.467	2.048	1.701	1.313
29	2.756	2.462	2.045	1.699	1.311
30	2.750	2.457	2.042	1.697	1.310
31	2.744	2.453	2.040	1.696	1.309
32	2.738	2.449	2.037	1.694	1.309
34	2.728	2.441	2.032	1.691	1.307
36	2.719	2.434	2.028	1.688	1.306
38	2.712	2.429	2.024	1.686	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
55	2.668	2.396	2.004	1.673	1.297
60	2.660	2.390	2.000	1.671	1.296
65	2.654	2.385	1.997	1.669	1.295
70	2.648	2.381	1.994	1.667	1.294
75	2.643	2.377	1.992	1.665	1.293
80	2.639	2.374	1.990	1.664	1.292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
750	2.582	2.331	1.963	1.647	1.283
1000	2.581	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.646	1.282
Large	2.576	2.326	1.960	1.645	1.282