Prob & Stats Exam 1 Summary & Review

Wednesday, February 5

This exam consists of 7 problems (15 parts) and is worth 80 points.

- You will be allowed to use a calculator on the exam to help with arithmetic. However, you will not be able to use any built-in probability or statistics functions of that calculator.
- You will be allowed to use the Help Page attached to this review
- You will be expected to show all of your work on most problems. Partial credit will be given on most problems, but only for proper work which is shown.

Topics

General information about Probability and Statistics
- What are they?
- What are the different categories of statistics?

Important definitions (e.g. random sample, sample space, event, outcome, rv, pdf, cdf)

Counting the size of a sample space
- Fundamental counting principle
- Permutations
- Combinations
- Variations and mixtures of these themes

Axioms of probability

Theorems of probability

Probabilities involving equally likely outcomes

Conditional probability

Addition rule and multiplication rules

Bayes Theorem

Independent events

Working with rv's , pdfs, & cdfs.

Finding means, std devs, and expected values

Setting up and solving problems involving the common distributions.

Prob & Stats Exam 1 Help Page

Bernoulli distribution

$f(x) = p^x q^{1-x}$ x = 0, 1

0   e.w.

Binomial distribution

$f(x) = \binom{n}{x} p^x q^{n-x}$ x = 0, 1, 2, ... , n

0   e.w.

Negative binomial distribution

$f(x) = \binom{x+r-1}{r-1} p^r q^x$ x = 0, 1, 2, ...

0   e.w.

Geometric distribution

$f(x) = pq^x$ x = 0, 1, 2, ...

0   e.w.

Hypergeometric distribution

$f(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$ x = ?

0   e.w.

Poisson distribution

$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ x = 0, 1, 2, ...

0   e.w.

Bayes Theorem

$$P(B_r | A) = \frac{P(B_r)P(A | B_r)}{\sum_{all\,i} P(B_i)P(A | B_i)}$$

Mean/SD using expectations

$$\mu = E(X)$$

$$\sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$$

SD from data

$$\sigma = \sqrt{\frac{\sum (X-\mu)^2}{n}}$$

$$s = \sqrt{\frac{\sum (X-\bar{X})^2}{n-1}}$$

Prob & Stats Exam 1 Name:
Fall 2018 100 points

This part of the exam consists of 9 problems (15 parts) and is worth 100 points. You may use a calculator on this exam for assistance with arithmetic. However, you may not use any statistical functions on that calculator. Show all of your work in the space provided (where appropriate). Partial credit will be given on most problems, but only for proper work which is shown.

1. (6 points each) Suppose the random variable **X** has the following pdf:

$$f(x)=\begin{cases} \frac{1}{2^x} & \text{for } x=1,2,3 \\ \frac{1}{8} & \text{for } x=4 \\ 0 & \text{elsewhere} \end{cases}$$

Find the following:

a) P(**X** ≥ 2)
b) The CDF of the distribution.
c) The mean of the distribution.
d) The standard deviation of the distribution
e) E(X+1) i.e. the expected value using the above pdf & the function g(x) = x+1

2. (6 points each) Consider this problem:

Suppose we take a multiple choice test, consisting of 6 questions, each with four possible answers. Having neglected to study, we simply randomly choose answers without even reading the questions or answers. What is the probability that we will guess at least 5 correctly?

a) This problem fits one of our standard/common pdf types. Tell me which type it is, and what the parameters are for this pdf.
b) Use the pdf to find the probability.

3. (6 points each) Consider this problem:

Suppose we have a box that holds 4 red blocks and 6 green blocks. We're going to play a game where we draw a block out of the box, observe its color, and then put it back into the box. We continue doing this until the block we draw is red. The number of greens we draw before we finally draw a red is our "score" in this game. What is the probability our score is 2 or less?

a) This problem fits one of our standard/common pdf types. Tell me which type it is, and what the parameters are for this pdf.
b) Use the pdf to find the probability.

4. (4 points) Suppose a random sample gave us the following data:

22 29 25 28

Find the standard deviation of this data. (Show your work.)

5. (8 points) Suppose we know that 20% of ENC students are Math Majors. And suppose we know that 80% of Math Majors have IQs over 150, while the percentage is just 30% for the rest of the student body. Suppose we select an ENC student at random, and find they have an IQ above 150. What is the probability they are a Math Major? (Hint: Bayes Theorem)

6. (8 points) Eight ENC students are racing those green rental bicycles around the perimeter of the campus. The top three finishers will get invited to the Tour d'France. How many sets of 3 can be selected here?

7. (8 points) An exam has 6 multiple choice questions (with four possible answers on each) and 6 True/False questions. How many different "strings of answers" can someone give for the exam? (Assume no answer is left blank.)

8. (6 points) State our three axioms of probability.

a)
b)
c)

9. (8 points) Choose one of the following properties to prove.

a) Using just the Axioms of Probability, prove that P(Ø) = 0

b) Using just the Axioms of Probability, and the fact that P(Ø) = 0, prove that $P(C_1 \cup \ldots \cup C_n) = P(C_1) + \ldots + P(C_n)$ where $C_i \cap C_j = Ø$

c) Suppose f(x) is a discrete pdf. We know that $\sigma^2 = \Sigma (x-\mu)^2 f(x)$. Manipulate that summation to show that $\sigma^2 = \Sigma x^2 f(x) - \mu^2$.

d) Prove that $\binom{n}{k} = \binom{n}{n-k}$, and briefly explain why it is logical for those two things to be equal.