5.1

#1. a) $P(X=1 \text{ and } Y=1) = p(1,1) = 0.20$

b) $P(X \le 1 \text{ and } Y \le 1) = p(0,0)+p(0,1)+p(1,0)+p(1,1)$
$= 0.10 + 0.04 + 0.08 + 0.20 = 0.42$

c) At least 1 self service and 1 full service hose is in use.
Prob = 0.20+0.06+0.14+0.30 = 0.70

d) $f_X(x) =$ 0.16 for x=0
0.34 for x=1
0.50 for x=2
0 elsewhere

$f_Y(y) =$ 0.24 for y=0
0.38 for y=1
0.38 for y=2
0 elsewhere

$P(X \le 1) = 0.16 + 0.34 = 0.50$

e) To be independent, we must have $f_X(x)*f_Y(y) = f(x,y)$ for all ordered pairs (x,y). Since $f_X(0)*f_Y(0) = 0.16*0.24 = 0.0384$,
and $f(0,0) = 0.10$, we do not have this property.
The rv's are not independent.

#3. a) $f(1,1) = 0.15$
b) 0.08 + 0.15 + 0.10 + 0.07 = 0.40
c) A = the event $|X_1-X_2| \ge 2$
The ordered pairs in this event would be
(0,2) (0,3)
(1,3)
(2,0)
(3,0) (3,1)
(4,0) (4,1) (4,2)
The prob would be 0.04 + 0.00 + 0.04 + 0.05 + 0.00 + 0.03
+ 0.00 + 0.01 + 0.05
= 0.22
d) P(total=4) = 0.04 + 0.10 + 0.03 + 0.00 = 0.17
$P(\text{total} \ge 4)$ = 0.17 + (0.06+0.04+0.01) + (0.07+0.05) + 0.06 = 0.46.

5.1, continued

#4 a) $f(x) =$
0.19 for x=0
0.30 for x=1
0.25 for x=2
0.14 for x=3
0.12 for x=4
0 elsewhere

$E(X) = \Sigma xf(x) = 0*0.19 + 1*0.30 + 2*0.25 + 3*0.14 + 4*0.12$
$= 0 + 0.30 + 0.50 + 0.42 + 0.48 = 1.70$

b) $f(x) =$
0.19 for x=0
0.30 for x=1
0.28 for x=2
0.23 for x=3
0 elsewhere

c) To be independent, we would need $P(X_1=4, X_2=0)$ to equal $P(X_1=4)*P(X_2=0)$. But 0.00 does not equal 0.12*0.19, so we do not have independent rv's.

#12. Part b first) To find the marginal of X, we integrate our pdf w/r to y from 0 to infinity. That gives:

$f_X(x) = e^{-x}$ for $x>0$, 0 elsewhere

To find the marginal of Y, we integrate our pdf w/r to x from 0 to infinity. That gives:

$f_Y(x) = (1+y)^{-2}$ for $y>0$, 0 elsewhere

The rv's are not independent, because the marginals do not multiply to equal the joint pdf.

a) We integrate our marginal of X from 3 to infinity. That gives e^{-3} for our probability, which is 0.0498.

c) Since P(at least one of X or Y exceeds 3) is the complement of P(neither X nor Y exceed 3), and that event is the square $0<x<3$, $0<y<3$, we double integrate our joint pdf over that square, then subtract the answer from 1.

The integral gives: $(e^{-12}/4 - e^{-3}) - (1/4 - 1)$.

So our prob is $1/4 - e^{-12}/4 + e^{-3} = 0.250 - 0.0000 + 0.0498$
$= 0.2998$.