5.3-4

#37. The chart shows the probabilities of the different pairs of values

		X2	
X1	25	40	65
25	0.04	0.10	0.06
40	0.10	0.25	0.15
65	0.06	0.15	0.09

a) From the chart above we can determine the different $\overline{X}$ values, as well as the probabilities:

$\overline{X}$	Prob		
25.0	0.04		1.00
32.5	0.20		6.50
40.0	0.25		10.00
45.0	0.12		5.40
52.5	0.30		15.75
65.0	0.09		5.85
			44.50

Columns A & B show the sampling distribution.
Column D calculates $E(\overline{X}) = 44.5$
$\mu = 25*0.20 + 40*0.50 + 65*0.30$
$= 5.0 + 20.0 + 19.5 = 44.5$, the same as $E(\overline{X})$.

b) From the chart above we can determine the different s^2 values, as well as the probabilities:

s^2	Prob		
0.0	0.38		0.00
112.5	0.20		22.50
312.5	0.30		93.75
800.0	0.12		96.00
			212.25

Columns A & B show the sampling distribution.
Column D calculates $E(S^2) = 212.25$

$\sigma^2 = 25^2*0.20 + 40^2*0.50 + 65^2*0.30 - 44.5^2$
$= 125.0 + 800.0 + 1267.5 - 1980.25$
$= 212.25$, the same as $E(S^2)$.

#38

a) The possible values of T_o are 0, 1, 2, 3, 4.

The way to get a 0 is for X_1 and X_2 to both be zero, so the prob is 0.2*0.2 = 0.04.

The way to get a 1 is for X_1=0 and X_2=1, or X_1=1 and X_2=0. The prob is 0.2*0.5 + 0.5*0.2 = 0.1 + 0.1 = 0.2

For 2: 0.2*0.3 + 0.5*0.5 + 0.3*0.2 = 0.06 + 0.25 + 0.06 = 0.37

For 3: 0.5*0.3 + 0.3*0.5 = 0.15 + 0.15 = 0.30

For 4: 0.3*0.3 = 0.09

The distribution, then, is:

T_o	Prob
0.0	0.04
1.0	0.20
2.0	0.37
3.0	0.30
4.0	0.09
elsewhere	0.00

b & c) To get the mean and variance, we need E(X) and E(X^2).

T_o	Prob		X*f(x)		X^2 *f(x)
0.0	0.04		0.00		0.00
1.0	0.20		0.20		0.20
2.0	0.37		0.74		1.48
3.0	0.30		0.90		2.70
4.0	0.09		0.36		1.44
elsewhere	0.00				
		E(X) =	2.20	E(X^2) =	5.82

So μ = 2.2, and σ^2 = 5.82 - 2.2^2 = 5.82 - 4.84 = 0.98.

Notice that μ = the sum of the means of X_1 & X_2,
and σ^2 = the sum of the variances of X_1 & X_2.

#46.
a) The center of $\overline{X}$'s distribution is $\mu = 12$, and the standard deviation is $\sigma/\sqrt{n} = 0.04/4 = 0.01$

b) The center is $\mu = 12$, and the standard deviation is $\sigma/\sqrt{n} = 0.04/8 = 0.005$

c) It is more likely to be within 0.01 cm on the second sample, since that encompasses ±2 standard deviations, while the first is only 1 standard deviation.

#47. a) Since the population is normal, we know that $\overline{X} \sim ND(12, 0.01)$.
$P(11.99 \le \overline{X} \le 12.01) = P(-1 \le z \le 1) = 0.8413 - 0.1587 = 0.6826$

b) In this case, $\overline{X} \sim ND(12, 0.008)$.
$P(\overline{X} \ge 12.01) = P(z \ge 1.25) = 1 - 0.8944 = 0.1056$

#49. The population is ?D(6 min, 6 min).
a) We want to know the probability 40 of these random variables add up to 4 hrs 10 minutes or less (i.e. 250 minutes)
$P(\Sigma X \le 250) = P(\Sigma X/n \le 250/40) = P(\overline{X} \le 6.25)$. Since the sample size is large (more than 30), we know that $\overline{X}$ is approximately $ND(6, 6/\sqrt{40})$. (That SD is 0.949). $P(\overline{X} \le 6.25) = P(z \le 0.25/0.949) = P(z \le 0.26) = 0.6026$.

b) $P(\Sigma X \ge 260) = P(\overline{X} \ge 6.5) = P(z \ge 0.53) = 1 - 0.7019 = 0.2981$.

#50. The population is ?D(10000, 500).
a) Since the sample size is large, we know $\overline{X}$ is approx $ND(10000, 500/\sqrt{40})$. (That SD is 79.1.) $P(9900 \le \overline{X} \le 10200) = P(-1.26 \le z \le 2.53) = 0.9943 - 0.1038 = 0.8905$

b) Not without assuming the population was normal.